# MATHEMATICS

## COMPLEX NUMBERS_SYNOPSIS

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| • | **Square root of a complex number.** $\sqrt{a+ib}=\pm\left[\sqrt{\frac{\sqrt{a^2+b^2}+a}{2}}+i\sqrt{\frac{\sqrt{a^2+b^2}-a}{2}}\right]$ **If** $b>0$ <br> $=\pm\left[\sqrt{\frac{\sqrt{a^2+b^2}+a}{2}}-i\sqrt{\frac{\sqrt{a^2+b^2}-a}{2}}\right]$ **If** $b<0$ |
| • | **If** $z=rcis\theta$ **then i)** $\log z=\log r+i\theta$ <br> **ii)** $\log i=i\pi/2$ **iii)** $i^i=e^{-\pi/2}$ <br> **iv)** $\sin\left(\log i^i\right)=-1$ <br> **v) If** $\left(i^i\right)^i=\cos\theta+i\sin\theta$ **then** $\theta=-\pi/2$ <br> **vi) Logrithm of a complex number then log** $\log z=\log\|z\|+i\arg z$ **or** <br> $\log z=\log\|z\|+i\left(\arg z+2k\pi\right)$**,k is an integer** <br> **i.e.** $\log\left(a+ib\right)=\frac{1}{2}\log\left(a^2+b^2\right)+i\tan^{-1}\frac{b}{a},a\neq 0$ <br> **vii)** $\log\left(1+i\right)=\frac{1}{2}\log 2+i\pi/4$ |
| • | $A(z_1),B(z_2),C(z_3),D(z_4)$ **are complex numbers then condition for AB to be parallel to CD is** $\frac{z_2-z_1}{z_4-z_3}$ **is purely real.** |
| • | **Let** $z_1,z_2,z_3$ **be the affixes of points of A,B,C respectively in the argand diagram. then the angle between AB and AC is arg** $\left(\frac{z_3-z_1}{z_2-z_1}\right)$. |
| • | **Let** $z_1,z_2,z_3,z_4$ **be the affixes of points of A,B,C,D respectively in the argand diagram. then the angle between DC and BA is arg** $\left(\frac{z_3-z_4}{z_1-z_2}\right)$ . |
| • | **The area of triangle whose vertices are** $Z,iZ,Z+iZ$ **is** $\frac{1}{2}\|z\|^2$. |
| • | **The area of triangle whose vertices are** $z,\omega z,z+\omega z$ **is** $\frac{\sqrt{3}}{4}\|z\|^2$. |

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| • | **If $z_1, z_2, ........z_n$ be the vertices of a regular polygon of 'n' sides and $z_0$ be its centroid then $z_1^2 + z_2^2 + ....... + z_n^2 = nz_0^2$.** |
| • | **Area of the triangle with vertices $z_1, z_2$ and $z_3$ is $\left\lvert \Sigma \left[ \frac{(z_2 - z_3)\lvert z_1 \rvert^2}{4iz_1} \right] \right\rvert$ sq. units** |
| • | **Equilateral triangle: The triangle whose vertices are the points $z_1, z_2, z_3$ on the Argand plane is an equilateral triangle iff** $z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$ **(or)** $\frac{1}{z_1 - z_2} + \frac{1}{z_2 - z_3} + \frac{1}{z_3 - z_1} = 0$ |
| • | **If the complex numbers $z_1, z_2, z_3$ be the vertices of an equilateral triangle and if $z_0$ be the circumcentre of the triangle, then $z_1^2 + z_2^2 + z_3^2 = 3z_0^2$** |
| • | **If $z_1^2 - z_1z_2 + z_2^2 = 0$ then the origin and $z_1, z_2$ forms an equilateral triangle.** |
| • | **If $z_1$, $z_2$, $z_3$ are the vertices of an isosceles right angle triangle right angled at $z_2$ then $z_1^2 + z_2^2 + z_3^2 = 2z_2(z_1+z_3)$** |
| • | **If $z_1^2 + z_1z_2 + z_2^2 = 0$ then the origin and $z_1, z_2$ forms an isosceles triangle.** |
| • | **Condition for collinearity : The three points $z_1$, $z_2$ and $z_3$ will be collinear if there exists real a, b, c statisfying $az_1 + bz_2 + cz_3 = 0$ such that a+b+c = 0. Three points $z_1$, $z_2$, $z_3$ are collinear**<br>**1.if** $\begin{vmatrix} z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \\ z_3 & \bar{z}_3 & 1 \end{vmatrix}$ **= 0 2.** $\frac{z_3 - z_1}{z_2 - z_1}$ **is purely real**<br>**3.** $\arg(z_2 - z_1) = \arg(z_3 - z_1)$<br>**4. Three complex number are in A.P.** |
| • | **Equation of a line : Equation of a line through $z_1$ and $z_2$ is given by** $\frac{z - z_1}{z - z_2} = \frac{\bar{z} - \bar{z}_1}{\bar{z} - \bar{z}_2}$<br>**(or)**<br>$\begin{vmatrix} z & \bar{z} & 1 \\ z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \end{vmatrix}$ **= 0.** |
| • | **The general equation of a striaght line is $\bar{a}z + a\bar{z} + b = 0$ where b is a real number.** |
| • | **Complex Slope of a straight line If $A(z_1), B(z_2)$ are two points in the Argand Plane then the complex slope ( $\mu$ )of the straight line AB is given by** $\mu = \frac{z_1 - z_2}{\bar{z}_1 - \bar{z}_2}$ |
| • | **Angle between two lines :** |

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| | If $A(z_1), B(z_2), C(z_3), D(z_4)$ are four points in the Argand plane then the angle $\theta$ between the lines AB and CD is given by $\theta = \arg\left(\frac{z_1 - z_2}{z_3 - z_4}\right)$ |
| • | **Length of the perpendicular from a point to a line:** The length of the perpendicular from a point $z_1$ to the line $a\bar{z} + \bar{a}z + b = 0$ is given by $\frac{a\bar{z}_1 + \bar{a}z_1 + b}{2\lvert a\rvert}$ |
| • | **INCENTRE:** The incentre of the triangle (in the Argand plane), formed by $z_1, z_2, z_3$ is $\frac{az_1 + bz_2 + cz_3}{a+b+c}$ where $a = \lvert z_2 - z_3\rvert,\ b = \lvert z_3 - z_1\rvert,\ c = \lvert z_1 - z_2\rvert$ |
| • | **EXCENTRES:** The excentres of the triangle (in the Argand plane), formed by $z_1, z_2, z_3$ are given by, ( i ) $I_1 = \frac{-az_1 + bz_2 + cz_3}{-a+b+c}$ ( ii ) $I_2 = \frac{az_1 - bz_2 + cz_3}{a-b+c}$ (iii) $I_3 = \frac{az_1 + bz_2 - cz_3}{a+b-c}$ where $a = \lvert z_2 - z_3\rvert,\ b = \lvert z_3 - z_1\rvert,\ c = \lvert z_1 - z_2\rvert$ |
| • | Square: If $z_1, z_2, z_3, z_4$ are the vertices of a square in that order, then a) $z_1 + z_3 = z_2 + z_4$ b) $\lvert z_1 - z_2\rvert = \lvert z_2 - z_3\rvert = \lvert z_3 - z_4\rvert = \lvert z_4 - z_1\rvert$ c) $\lvert z_1 - z_3\rvert = \lvert z_2 - z_4\rvert$ d) $\frac{(z_1 - z_3)}{(z_2 - z_4)}$ is purely imaginary. |
| • | The least value of $\lvert z-a\rvert + \lvert z-b\rvert$ is $\lvert a-b\rvert$ |
| • | If P represents a complex number 'z' in the argand diagram and OP is rotated through an angle $\alpha$ and Q is the new position of 'P' then the complex number represented by Q is $z.cis\alpha$ . |
| • | The equation $\left\lvert\frac{z - z_1}{z - z_2}\right\rvert = k, k = 1$ represents the perpendicular bisector of line segment joining $z_1, z_2$ |
| • | The equation $\left\lvert\frac{z - z_1}{z - z_2}\right\rvert = k, k \neq 1$ represents a circle |
| • | The equation of a circle having centre $z_0$ and radius $r$ is $\lvert z - z_0\rvert = r \ or \ z\bar{z} - z_0\bar{z} - \bar{z}_0 z + z_0\bar{z}_0 - r^2 = 0$ |

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| • | **The equation of the circle described on the line segment joining** $z_1$ **and** $z_2$ **as Diameter is** $(z-z_1)(\bar{z}-\bar{z}_2)+(z-z_2)(\bar{z}-\bar{z}_2)=0$ **(or)** $\lvert z-z_1\rvert^2+\lvert z-z_2\rvert^2=\lvert z_1-z_2\rvert^2$ |
| • | **The general equation of a circle is** $z\bar{z}+a\bar{z}+\bar{a}z+b=0$ **where b is a real number. The centre of the circle is '- a' and its radius is** $\sqrt{a\bar{a}-b}$. |
| • | $\lvert z-z_1\rvert^2+\lvert z-z_2\rvert^2=K \quad (K\in R)$ **will represent a circle iff** $K\geq\frac{1}{2}\lvert z_1-z_2\rvert^2$ |
| • | $\lvert z-z_1\rvert+\lvert z-z_2\rvert=k$ **represents**<br>**1. an ellipse if** $k>\lvert z_1-z_2\rvert$<br>**2. an empty set if** $k<\lvert z_1-z_2\rvert$<br>**3. a line segment if** $k=\lvert z_1-z_2\rvert$ |
| • | $\lvert z-z_1\rvert-\lvert z-z_2\rvert=k$ **represents**<br>**1. a hyperbola if** $k<\lvert z_1-z_2\rvert$<br>**2. an empty set if** $k>\lvert z_1-z_2\rvert$<br>**3. two rays if** $k=\lvert z_1-z_2\rvert$ |
| • | $\lvert z-z_1\rvert^2+\lvert z-z_2\rvert^2=\lvert z_1-z_2\rvert^2$ **is a circle**<br>**(with** $z_1$ **and** $z_2$ **as ends of a diameter)** |
| • | $\arg\left(\frac{z-z_1}{z-z_2}\right)=a$ **fixed angle, is a part of the circle.** |
| • | **Concyclic points: Four points** $z_1, z_2, z_3$ *and* $z_4$ **are concyclic if and only if** $\frac{(z_1-z_3)(z_2-z_4)}{(z_1-z_4)(z_2-z_3)}$ **is purely real** |
| • | **The equation of a line perpendicular to the line** $a\bar{z}+\bar{a}z+b=0$ **is** $a\bar{z}-\bar{a}z+i\lambda=0$, **where** $\lambda\in R$. |
| • | **The length of the perpendicular from a point P ($z_0$) to the line** $a\bar{z}+\bar{a}z+b=0$ **is** $\left\lvert\frac{a\bar{z}_0+\bar{a}z_0+b}{2\lvert a\rvert}\right\rvert$. |
| • | **Lines** $\alpha\bar{z}-\bar{\alpha}z+\lambda=0$ **and** $\beta\bar{z}-\bar{\beta}z+\mu=0$ **are mutually perpendicular, if** $\alpha\bar{\beta}+\bar{\alpha}\beta=0$. i.e. $\mathrm{Re}(\alpha\bar{\beta})=0$. |
| • | **If n is an integer, then** $(\cos\theta+i\sin\theta)^n=\cos n\theta+i\sin n\theta$ |
| • | **If** $\frac{p}{q}$ **(q>1) is a rational number then** $\cos\frac{p\theta}{q}+i\sin\frac{p\theta}{q}$ **is one root of the $q^{th}$ roots of** $(\cos\theta+i\sin\theta)^p$ |
| • | **$n^{th}$ root of a complex number :** |

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| | **The $n^{th}$ roots of a complex number** $z = rcis\theta$ **are** $r^{1/n}cis\left(\frac{2k\pi+\theta}{n}\right)$ <br> **where** $k = 0, 1, 2, .......n-1$ |
| • | $n^{th}$ **roots of unity :** <br> $n^{th}$ **roots of unity are** $1, \alpha, \alpha^2, ...., \alpha^{n-1}$, **where** <br> $\alpha = e^{\left(\frac{i2\pi}{n}\right)} = \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}$ <br> **Following are the properties of** $n^{th}$ **roots of unity:** <br> **i)** $n^{th}$ **roots of unity form a G.P. with common ratio** $\alpha = e^{i2\pi/n}$ <br> **ii) The sum of** $n^{th}$ **roots of unity is zero.** <br> **iii) The sum of** $p^{th}$ **powers of** $n^{th}$ **roots of unity is zero, if p is not a multiple of n.** <br> **iv) The sum of the** $p^{th}$ **powers of** $n^{th}$ **roots of unity is n, if p is a multiple of n.** <br> **v) The product of** $n^{th}$ **roots of unity is** $(-1)^{n-1}$ <br> **vi)** $n^{th}$ **roots of unity lie on the circle** $\|z\| = 1$ **and divides its circumference into n equal parts.** <br> **vii)** $n^{th}$ **roots of unity are the roots of the equation** $z = 1^{1/n}$ **i.e.** $z^n = 1$ **or** $z^n - 1 = 0$ <br> **viii)** $n^{th}$ **roots of –1 are the roots of the equation** $z^n + 1 = 0$. <br> **ix) The arguments of the numbers are in A.P. with common difference** $\frac{2\pi}{n}$. <br> **x) The length of side of polygon =** $\|\alpha^i - \alpha^{i+1}\| = 2\sin\left(\frac{\pi}{n}\right)$ <br> **xii) The area of polygon =** $\frac{n}{2}\sin\left(\frac{2\pi}{n}\right)$ <br> **xiii)** $z^n - 1 = 0$ **is a reciprocal equation i.e. replacing z by** $\frac{1}{z}$ **we get the same equation. Thus whenever** $\alpha$ **is a root,** $1/\alpha$ **is also a root.** <br> **xiv) Thus** $z^n - 1 = 0$ **has also roots** $1, \frac{1}{\alpha}, \frac{1}{\alpha^2}, \frac{1}{\alpha^3}, ...., \frac{1}{\alpha^{n-1}}$. |
| • | **If** $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$ **then** <br> **i)** $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha+\beta+\gamma)$ <br> **ii)** $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha+\beta+\gamma)$ <br> **iii)** $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = 0$ <br> **iv)** $\sin 2\alpha + \sin 2\beta + \sin 2\gamma = 0$ <br> **v)** $\cos 2^n\alpha + \cos 2^n\beta + \cos 2^n\gamma = 0$ <br> **vi)** $\sin 2^n\alpha + \sin 2^n\beta + \sin 2^n\gamma = 0$ <br> **vii)** $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 3/2$ |

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| | **viii)** $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=3/2$<br>**ix)** $\cos(2\alpha-\beta-\gamma)+\cos(2\beta-\gamma-\alpha)+\cos(2\gamma-\alpha-\beta)=3$<br>**x)** $\sin(2\alpha-\beta-\gamma)+\sin(2\beta-\gamma-\alpha)+\sin(2\gamma-\alpha-\beta)=0$<br>**Consider the equation in variable 'x', for convenience $x^n-1=0$, whose roots are n<sup>th</sup> roots of unity.**<br>**From algebra of polynomials.**<br>$x^n-1\equiv(x-1)(x-\alpha)(x-\alpha^2)(x-\alpha^3)....(x-\alpha^{n-1})$<br>**(or)**<br>$x^n-1=(x-1)(x-\alpha_1)(x-\alpha_2)(x-\alpha_3).....(x-\alpha_{n-1})$<br>$\Rightarrow\frac{x^n-1}{x-1}\equiv(x-\alpha_1)(x-\alpha_2)(x-\alpha_3).....(x-\alpha_{n-1})$<br>**This is an identity.**<br>**i) Put x = 1,** $(1-\alpha_1)(1-\alpha_2).....(1-\alpha_{n-1})=n$<br>**ii) Put x=2,** $(2-\alpha_1)(2-\alpha_2).....(2-\alpha_{n-1})=2^n-1$<br>**iii) Put x=3,** $(3-\alpha_1)(3-\alpha_2)...(3-\alpha_{n-1})=\frac{3^n-1}{2}$<br>**iv) Put x = –1,** $(1+\alpha_1)(1+\alpha_2).....(1+\alpha_n)=\frac{(-1)^n-1}{-2(-1)^{n-1}}=\frac{1}{2}\{1-(-1)^n\}$<br>**v) Put x = –2,** $(2+\alpha_1)(2+\alpha_2).....(2+\alpha_n)=\frac{1}{3}\{2^n-(-1)^n\}$<br>**vi) Put x = i,** $(i-\alpha_1)(i-\alpha_2).....(i-\alpha_{n-1})=\frac{i^n-1}{i-1}$<br>**vii) Put,** $x=\omega$, $(\omega-\alpha_1)(\omega-\alpha_2)....(\omega-\alpha_{n-1})=\frac{\omega^n-1}{\omega-1}$<br>**viii) Put x = 1, –1 and multiply,** $(1-\alpha_1^2)(1-\alpha_2^2)....(1-\alpha_{n-1}^2)=\frac{n}{2}\{1-(-1)^n\}$ |
| • | **If** $\left\|z+\frac{1}{z}\right\|=a$, **the greatest and least values of** $\|z\|$ **are respectively** $\frac{a+\sqrt{a^2+4}}{2}$ **and** $\frac{-a+\sqrt{a^2+4}}{2}$. |
| • | $\left\|z_1+\sqrt{z_1^2-z_2^2}\right\|+\left\|z_2-\sqrt{z_1^2-z_2^2}\right\|=\|z_1+z_2\|+\|z_1-z_2\|$ |

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| • | **If** $z_1 = z_2 \Leftrightarrow \lvert z_1\rvert = \lvert z_2\rvert$ **or** $\arg z_1 = orgz_2$ |
| • | $\lvert z_1 + z_2\rvert = \lvert z_1\rvert + \lvert z_2\rvert \Leftrightarrow \arg(z_1) = \arg(z_2)$ **i.e.,** $z_1$ **and** $z_2$ **are parallel.** |
| • | $\lvert z_1 + z_2\rvert = \lvert z_1\rvert + \lvert z_2\rvert \Leftrightarrow \arg(z_1) - \arg(z_2) = 2n\pi$, **where n is some integer.** |
| • | $\lvert z_1 - z_2\rvert = \lvert\lvert z_1\rvert - \lvert z_2\rvert\rvert \Leftrightarrow \arg(z_1)\arg(z_2) = 2a\pi$, **where n is some integer.** |
| • | $\lvert z_1 + z_2\rvert = \lvert z_1 - z_2\rvert \Leftrightarrow \arg(z_1) - \arg(z_2) = \pi / 2$. |

## COMPLEX NUMBERS_ASSIGNMENT

1. The least positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$ is

1) 1 2) 2 3) 3 4) 4

2. If $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = a + ib$ then (a, b) =

1) (0, 2) 2) (0, -2) 3) (2, 0) 4) (0, 1)

3. The conjugate of $\frac{2-i}{(1-2i)^2}$ is

1) $\frac{-2+11i}{25}$ 2) $\frac{-2-11i}{25}$ 3) $\frac{2-11i}{25}$ 4) $\frac{2+11i}{25}$

4. If $x + iy = \sqrt{\frac{a+ib}{c+id}}$ then $(x^2 + y^2)^2 =$

1) $\frac{a^2-b^2}{a^2+b^2}$ 2) $\frac{a^2+b^2}{c^2-d^2}$ 3) $\frac{a^2+b^2}{c^2+d^2}$ 4) $\frac{a^2-b^2}{c^2-d^2}$

5. If (1–i)(1–2i)(1–3i)…..(1–10i) = x –iy, then 2.5.10….101 =

1) x + y 2) $x^2 + y$ 3) $x + y^2$ 4) $x^2 + y^2$

6. If $x + iy = \frac{1}{1+\cos\theta + i\sin\theta}$, then x =

1) 1/4 2) 1/2 3) 1 4) 1/3

7. If $x = 2 - i\sqrt{7}$, then $3x^3 - 4x^2 + x =$

1) -88 2) 0 3) 88 4) 1

8. $\text{Amp}\left(\frac{1+i\sqrt{3}}{-1-i}\right) =$

1) $195^0$ 2) $165^0$ 3) $-165^0$ 4) $215^0$

9. The amplitude of $(-1)^5$ is

1) $3\pi/2$ 2) $\pi$ 3) $\pi/2$ 4) $\pi/4$

10. log (log i) =

1) $\log\left(\frac{\pi}{2} + i\frac{\pi}{2}\right)$ 2) $\log\left(\frac{\pi}{2} - i\frac{\pi}{2}\right)$ 3) $\log\frac{\pi}{2} + i\frac{\pi}{2}$ 4) $\log\frac{\pi}{2} - i\frac{\pi}{2}$

11. $\tan\left[i\log\left(\frac{a-ib}{a+ib}\right)\right] =$

1) ab 2) $\frac{2ab}{a^2-b^2}$ 3) $\frac{a^2-b^2}{2ab}$ 4) $\frac{2ab}{a^2+b^2}$

12. The locus of Z satisfying $\left|\frac{Z-4-3i}{2i(Z-1)}\right| = 1$ is

1) a line 2) circle

3) a circle with centre i 4) a circle with centre -i

13. If $z_1$, $z_2$ are complex numbers and $z_1/z_2$ is purely imaginary, then $\left|\frac{z_1+z_2}{z_1-z_2}\right|$ is equal to

1) 1 2) 2 3) 3 4) 4

14. The minimum value of $|Z| + |Z-1|$ is

1) 0 2) 1 3) -1 4) 2

15. If $\left|z-\frac{4}{z}\right|=2$, then the greatest value of |Z| is

1) $\sqrt{5}-1$ 2) $\sqrt{5}$ 3) $\sqrt{5}+1$ 4) $\sqrt{5}+2$

16. If the expression $\frac{1+2i\cos\theta}{1-i\cos\theta}$ is a real number, the real value of $\theta$ is

1) $2n\pi \pm (\pi/4)$ 2) $2n\pi \pm (\pi/2)$ 3) $2n\pi \pm (\pi/3)$ 4) $n\pi \pm \pi/2$

17. In the argand plane, if three vertices of a square are represented by the complex numbers 3-i, -2-2i and 4-6i, then its fourth vertex is represented by

1) 9-5i 2) –1-7i 3) –3+3i 4) 9-9i

18. If z = x-iy and $z^{\frac{1}{3}} = p+iq$, then $\frac{\left(\frac{x}{p}+\frac{y}{q}\right)}{p^2+q^2}$ is equal to

1) 1 2) –2 3) 2 4) –1

19. The real and imaginary parts of $\cos\left(\frac{\pi}{4}+i\log 2\right)$ are

1) $\left(\frac{5}{4\sqrt{2}},\frac{3}{4\sqrt{2}}\right)$ 2) $\left(\frac{-5}{4\sqrt{2}},\frac{-3}{4\sqrt{2}}\right)$ 3) $\left(\frac{-5}{4\sqrt{2}},\frac{3}{4\sqrt{2}}\right)$ 4) $\left(\frac{5}{4\sqrt{2}},\frac{-3}{4\sqrt{2}}\right)$

20. The region represented by |Z – a| + |Z + a| > 4a is

1) area outside the circle $x^2+y^2 = 16a^2$

2) area inside the circle $x^2+y^2 = 16a^2$

3) area to the left of x = 4a

4) area out side the curve $\frac{x^2}{4a^2}+\frac{y^2}{3a^2}=1$

21. If P, Q represent $Z_1$ , $Z_2$ in the Argand diagram and $\angle POQ = 90^0$ then $Z_1\bar{Z}_2+\bar{Z}_1.Z_2$

1) 0 2) 1 3) $Z_1$ $Z_2$ 4) $\bar{Z}_1$

22. If $z_1$, $z_2$, $z_3$ are vertices of an equilateral triangle inscribed in a circle |z| = 2 and if $z_1=1+i\sqrt{3}$,

then $z_2, z_3$ are

1) -2, $1 - i\sqrt{3}$ 2) -2, $-1-i\sqrt{3}$ 3) 2, $1 - i\sqrt{3}$ 4) 2, $1+i\sqrt{3}$

23. If $|Z_1| = |Z_2|=|Z_3| =\ldots\ldots=|Z_n|=1$, then $|Z_1+Z_2+Z_3+\ldots\ldots+Z_n| =$

1) $\left|\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}+\ldots..+\frac{1}{Z_n}\right|$ 2) $\left|\frac{1}{Z_1}-\frac{1}{Z_2}+\frac{1}{Z_3}-\ldots\ldots+\frac{1}{Z_n}\right|$

3) $\left|\frac{1}{Z_2{}^2}+\frac{1}{Z_2{}^2}+\frac{1}{Z_3{}^2}+\ldots\ldots+\frac{1}{Z_n{}^2}\right|$ 4) $|Z_1 - Z_2 + Z_3 + \ldots.. + Z_n|$

24. If $|z| = 3$, the area of the triangle whose sides are modulii of z, ωz, z+ωz (ω is a complex cube root of unity) is

1) $\frac{9\sqrt{3}}{4}$ 2) $\frac{3\sqrt{3}}{4}$ 3) $\frac{8\sqrt{3}}{4}$ 4) $\frac{6\sqrt{3}}{4}$

25. The roots $z_1, z_2, z_3$ of the equation $x^3 + 3ax^2 + 3bx + c = 0$ (a,b,c ε C) form an equilateral triangle in the argand plane if

1) $a^2 = b$ 2) $a = \pm b$ 3) $a = b^2$ 4) $|a| = |b|$

26. Let $z_1$ and $z_2$ be roots of the equation $z^2 + pz + q = 0$ where coefficients p and q may be complex numbers. Let A and B represent $z_1$ and $z_2$ in the complex plane.

If $\angle AOB = \alpha \neq 0$ and OA = OB where O is the origin then $4q\cos^2\frac{\alpha}{2} =$

1) p 2) $p^2$ 3) $p^3$ 4) $p^4$

27. The origin and the complex numbers represented by the roots of the equation $z^2 + az + b = 0$ form an equilateral triangle. Then

1) a = 3b 2) $a^2 = b$ 3) $a^2 = 3b$ 4) b = 3a

28. If $i^{i^{i^{\cdot^{\cdot^{\cdot^{\infty}}}}}}$ = x+iy then $\tan\frac{\pi x}{2}$

1) $\frac{x}{y}$ 2) $\frac{\pi y}{2}$ 3) $\frac{y}{x}$ 4) $\frac{-y}{x}$

29. The locus of z which satisfies the inequality $\log_{\frac{1}{5}}|z-5i|<\log_{\frac{1}{5}}|z+5i|$ is given by

1) $x \le 0$ 2) $x > 0$ 3) $y < 0$ 4) $y > 0$

30. The real part of cosh (α + iβ) + sinh (α + i β) is

1) $e^{\alpha}$ 2) $e^{\alpha}\cos\alpha$ 3) $e^{\alpha}\cos\beta$ 4) $2e^{\alpha}\cos\beta$

31. Let z, ω be complex numbers such that $\bar{z}+i\bar{\omega}=0$ and arg zω= π. Then arg z equals

1) π/4 2) 5π/4 3) 3π/4 4) π/2

32. I : If $z=\bar{z}$ the z is purely imaginary

II : If $z=-\bar{z}$ then z is purely real

1) only I is true 2) only II is true

3) both I& II are true 4) neither I nor II are true

33. A : If $2z_1-3z_2+z_3=0$, then $z_1, z_2, z_3$ are collinear points

R: If a, b, c ∈ R such that a + b + c = 0 and $az_1+bz_2+cz_3=0$, then $z_1, z_2, z_3$ are collinear

1) A is true, R is true and R is correct explanation of A

2) A is true, R is true and R is correct explanation of A

3) A is true, R is false

4) A is false, R is true

34. A : The origin and the roots of the equation $x^2+ax+b=0$ form an equilateral triangle if $a^2=3b$

R : If $z_1, z_2, z_3$ are vertices of an equilateral triangle then $z_1^2+z_2^2+z_3^2=z_1z_2+z_2z_3+z_3z_1$

1) A is true, R is true and R is correct explanation of A

2) A is true, R is true and R is correct explanation of A

3) A is true, R is false

4) A is false, R is true

35. Match the following

z is a complex number Locus of z

| | |
|---|---|
| I. $\lvert z \rvert = 1$ | a) straight line |
| II. $\lvert z+2i \rvert + \lvert z-2i \rvert = 4$ | b) ellipse |
| III. $Re(z^2)=4$ | c) hyperbola |
| IV. $z + \bar{z} = 4$ | d) pair of straight lines |
| | e) circle |

1) b, c, a, d 2) e, c, a, d 3) e, b, c, a 4) a, b, c, d

# KEY SHEET

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1) | 4 | 2) | 2 | 3) | 1 | 4) | 3 | 5) | 4 | 6) | 2 | 7) | 1 |
| 8) | 3 | 9) | 2 | 10) | 3 | 11) | 2 | 12) | 4 | 13) | 1 | 14) | 2 |
| 15) | 3 | 16) | 2 | 17) | 1 | 18) | 2 | 19) | 4 | 20) | 4 | 21) | 1 |
| 22) | 1 | 23) | 1 | 24) | 1 | 25) | 1 | 26) | 2 | 27) | 3 | 28) | 3 |
| 29) | 3 | 30) | 3 | 31) | 3 | 32) | 4 | 33) | 1 | 34) | 1 | 35) | 3 |